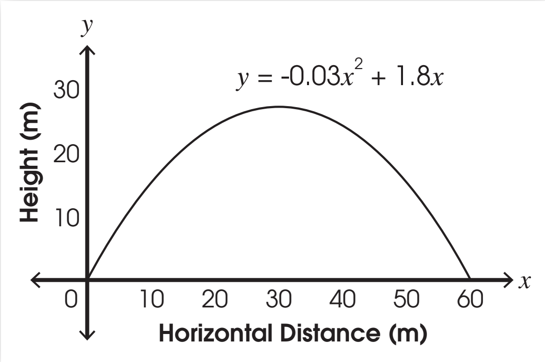
Quadratic equations are extremely useful. You have learned to determine the key features, such as the vertex and x-intercepts, of a quadratic equation. Have you ever considered how important it is to identify these key features? In architecture and civil engineering, quadratic equations are utilized to build and model equipment and structures.

Roller Coasters

If you are a thrill-seeker, roller coasters may appeal to you because of the excitement they offer with their steep slopes, tight twists, and inversions. Roller coasters arc up and down and sometimes around, very often resembling the shape of a parabola. To design a roller coaster, architects make use of quadratic relations to determine the height of the peaks and dips, which are the maximums and minimums of parabolas, that the roller coaster will reach.



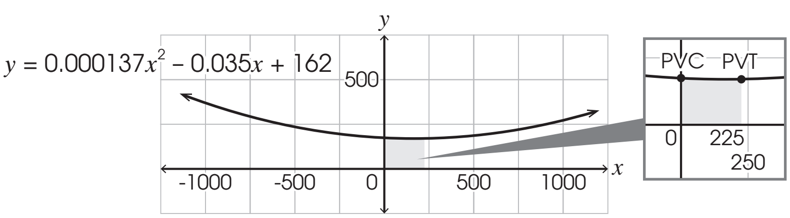
To better understand how quadratic equations can be applied in the design of a roller coaster, consider this simple example. A roller coaster to be built would employ a railroad track with hills, helixes, and loops. The shape of one of the hills would resemble the shape of a parabola and can be approximated and modelled by $y = -0.03x^2 + 1.8x$, where y is the height in metres and x is the horizontal distance travelled in metres. Using this equation, it can be determined that after the roller coaster travels a horizontal distance of 30 m from the starting point of this hill, it will reach the peak at the height of 27 m. This piece of information along with a variety of other computations and data, such as air resistance, gravity, and inertia, would allow architects to design a safe ride while providing an exciting and exhilarating experience for thrill-seekers.

Road Constructions

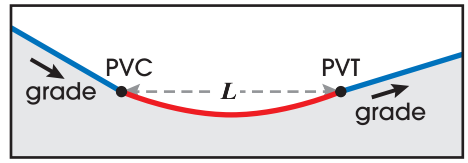
If you have ever biked on a steep slope, you know how exhausting it is biking uphill and how effortless it is going downhill. When it comes to road constructions, it might seem obvious and straightforward to determine how steep slopes should be, but this is not the case; rather, the steepness of slopes must be carefully assessed to ensure functionality, feasibility, and most importantly, road safety.

To create a smooth transition to connect two straight roads, a curve must be created. Consider the diagram on the right which illustrates the cross-section of a road. The blue lines represent the straight sections of the road and there is a gap between them. The red curve represents a curved road which would be built to bridge the gap. This curve is parabolic and civil engineers use quadratic equations to determine the shape of this curve.

If a road has a grade of -3.5% downhill, a grade of 2.7% uphill, with L being 225 m, and the elevation of PVC being 162 m, then the parabolic curve can be modelled by $y = 0.000137x^2 - 0.035x + 162$, where x denotes the distance from the PVC in metres and y is the elevation of the point on the curve in metres. The parabola of the equation is shown below.



Note that the horizontal distance between PVC and PVT is 225 m, as indicated by L. As a result, any value of x along the curve from 0 to 225 can be employed. Any x-value that falls outside this range is regarded as meaningless.



To determine the quadratic equation, civil engineers need the information below.

- **grade:**
  the steepness of a section of a road; a flat road has a grade of 0%, an uphill road has a positive grade (e.g. 5.2%), and a downhill road has a negative grade (e.g. -2.9%)
- **point of vertical curvature (PVC):**
  the point where a straight road ends and the curve begins
- **point of vertical tangency (PVT):**
  the point where the curve ends
- ***L*:**
  a variable that represents the horizontal distance from PVC to PVT, measured in metres

The roller coaster and road construction are only two examples of how quadratic equations are used by architects and engineers. Basically, a quadratic equation may describe anything that can be represented by a parabola. It can be applied to areas including designs, constructions, movements, and trajectories, among many other. The number of applications quadratic equations can be applied to is endless.