

## 2.5 Radians

A degree (abbreviated as  $^\circ$ ) is a measuring unit for angles. Although you are probably familiar with using degrees to describe the size of an angle, angles are generally measured in radians in most mathematical work outside of practical geometry.

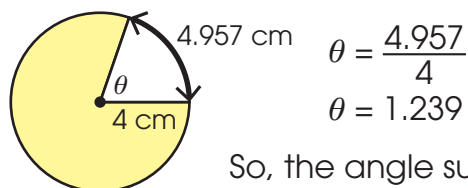
The radian is a unit used to measure angles. The radian system is commonly used in angle measurements in more advanced mathematics. In most fields of mathematics and science, such as calculus and physics, angles are measured in radians. In fact, the radian is an SI unit while the degree is not.

### Radian System

The radian is denoted by “rad”. One radian is defined as the angle formed at the centre of a circle by an arc whose length equals the radius of the circle and 1 rad is approximated to be  $57.296^\circ$ . Generally, when an angle is written without a unit, it is understood that the angle is expressed in radians.

To determine the radian of an angle, the arc length formed by the two arms should be known along with the radius of the circle.

For a circle with a radius of 4 cm, the angle in radian subtended by a 4.957-cm arc can be found as follows:



So, the angle subtended by the two arms is 1.239 rad.

Now, consider a full circle. We know that in degrees, its angle is  $360^\circ$ . In radians, the arc length, which is also the circumference, is  $2\pi r$ , meaning that the angle is  $\frac{2\pi r}{r} = 2\pi$ . Therefore,  $2\pi \text{ rad} = 360^\circ$ . Knowing this formula also allows us to do conversions between radians and degrees.

#### Radians to Degrees

$$2\pi \text{ rad} = 360^\circ$$

$$1 \text{ rad} = \frac{360^\circ}{2\pi}$$

$$1 \text{ rad} = \frac{180^\circ}{\pi}$$

Thus, to convert from radians to degrees, multiply the angle by  $\frac{180^\circ}{\pi}$ .

e.g. Convert  $\frac{\pi}{4}$  to degrees.

$$\frac{\pi}{4} \times \frac{180^\circ}{\pi} = \frac{180^\circ}{4} = 45^\circ$$

#### Degrees to Radians

$$360^\circ = 2\pi \text{ rad}$$

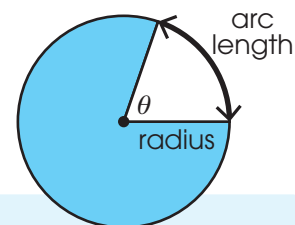
$$1^\circ = \frac{2\pi}{360} \text{ rad}$$

$$1^\circ = \frac{\pi}{180} \text{ rad}$$

Thus, to convert from degrees to radians, multiply the angle by  $\frac{\pi}{180}$  rad.

e.g. Convert  $45^\circ$  to radians.

$$45 \times \frac{\pi}{180} = \frac{\pi}{4}$$



#### Radian Formula

$$\theta = \frac{\text{arc length}}{\text{radius}}$$

↑  
angle subtended by arc

Degree	Radian (exact)	Radian (approx.)
$30^\circ$	$\frac{\pi}{6}$	0.524 ← $\frac{3.1416}{6}$
$45^\circ$	$\frac{\pi}{4}$	0.785 ≈ 0.524
$60^\circ$	$\frac{\pi}{3}$	1.047
$90^\circ$	$\frac{\pi}{2}$	1.571
$180^\circ$	$\pi$	3.142
$270^\circ$	$\frac{3\pi}{2}$	4.712
$360^\circ$	$2\pi$	6.283