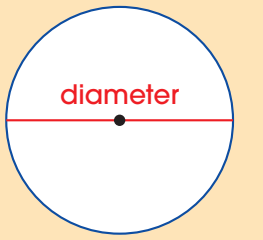
Any real number is either a rational number or an irrational number. As you have studied in Chapter 1.2, rational numbers are numbers that can be expressed as a fraction of two integers. This implies that irrational numbers are numbers that cannot be expressed as a fraction of integers. In decimal form, irrational numbers are non-terminating and non-repeating, meaning that their decimal part goes on infinitely and that their decimal digits have no repeating patterns. There are many irrational numbers. For example, the square root of any non-perfect square is an irrational number (e.g. $\sqrt{2}$ and $\sqrt{101}$). In fact, there are infinitely many irrational numbers but there are just a few irrational numbers that are special. Due to the significance and influence of these unique irrational numbers, names and symbols are given to represent them.

Pi (Symbol: π, representing 3.1415926535...)

π is the ratio of a circle's circumference to its diameter. It is a constant that applies to all circles. The symbol "π" is actually a Greek letter and is the first letter in the Greek word for circumference. Historically, mathematicians from many different cultures made progress toward defining this ratio. Ancient Egyptians approximated the ratio to be $\frac{22}{7}$, while Babylonians chose $\frac{25}{8}$. In ancient China, the number was once thought to be $\sqrt{10}$, while in India it was $\frac{339}{108}$. Regardless, they were all attempts to approximate the value of π, a representation of this unique ratio that we know of today.

π

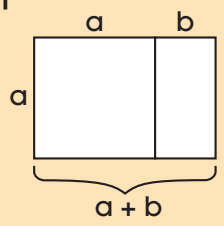


$$\pi = \frac{\text{circumference}}{\text{diameter}}$$

Golden Ratio (Symbol: φ, representing 1.6180339887...)

The golden ratio, φ, is a unique mathematical relationship. Two quantities are in the golden ratio if the ratio of these two quantities (a, b) is the same as the ratio of the sum of the two quantities (a + b) to the larger quantity (a). φ appears frequently in patterns in plants and animals. For example, the spiral on a snail shell and the number of petals on a lily are often related to the golden ratio. Because of its relation to elegant patterns in nature, many artists and architects throughout history, including Leonardo da Vinci and Salvador Dali, have employed the golden ratio in their works.

φ



$$\varphi = \frac{a}{b} = \frac{a+b}{a}$$

Euler's Number (Symbol: e, representing 2.7182818284...)

Euler's number, e, is named after a Swiss mathematician Leonhard Euler, who introduced e to represent the base for natural logarithm, even though it was Jacob Bernoulli who discovered the constant value that e represents while he was studying compound interest. e is a unique number; the logarithm (log) of any number to the base of e is specifically referred to as the natural logarithm (ln) of the number (i.e. $\log_e x = \ln x$). It is an important constant in mathematics and e has broad real-world applications. It is used in calculating growth and change over time in statistics, and in finding compound interest in the field of finance, for example.