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Mathematics of Data Management

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Test 2

Derivatives and Their Applications

Test Areas

- finding derivatives of polynomial functions
- understanding and applying derivative rules
- determining the maximum and minimum of a function
- solving optimization and other application problems

Knowledge and Understanding

Circle the correct answers.

① Which of the following is the notation of a derivative for $y = f(x)$?

A. $f'(x)$ B. y'

C. $\frac{dy}{dx}$ D. all of the above

② Which is the definition of a derivative?

A. $\lim\limits_{h \to 0} \frac{f(a + h) - f(a)}{h}$

B. $\lim\limits_{x \to a} \frac{f(x) - f(a)}{x - a}$

C. $\lim\limits_{x \to 0} \frac{f(x + h) - f(x)}{x + h}$

D. A and B

③ If $f(x) = k$, where k is a constant, then

A. $f'(x) = k$. B. $f'(x) = 1$.

C. $f'(x) = 0$. D. $f'(x) = x$.

④ If $f(x) = x^n$, where n is a real number, then

A. $f'(x) = nx$. B. $f'(x) = nx^{-1}$.

C. $f'(x) = nx^n$. D. $f'(x) = nx^{n-1}$.

⑤ The constant multiple rule states that if $f(x) = kg(x)$, where k is a constant, then

A. $f'(x) = kg(x)$. B. $f'(x) = kg'(x)$.

C. $f'(x) = k + g(x)$. D. $f'(x) = g(x)g'(x)$.

⑥ Name the following rule: if $f(x) = p(x) - q(x)$, then $f'(x) = p'(x) - q'(x)$.

A. the sum rule B. the difference rule

C. the product rule D. the quotient rule

⑦ Which of the following is the product rule if $h(x) = f(x)g(x)$?

A. $h'(x) = f'(x)g'(x)$

B. $h'(x) = f'(x)g(x) + f(x)g'(x)$

C. $h'(x) = f'(x)g(x) - f(x)g'(x)$

D. $h'(x) = f'(x)g'(x) + f(x)g(x)$

⑧ Which of the following is the quotient rule if $h(x) = \frac{f(x)}{g(x)}$?

A. $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$, $g(x) \neq 0$

B. $h'(x) = \frac{f(x)g'(x) - f'(x)g(x)}{[g(x)]^2}$, $g(x) \neq 0$

C. $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[f(x)]^2}$, $f(x) \neq 0$

D. $h'(x) = \frac{f(x)g'(x) - f'(x)g(x)}{[f(x)]^2}$, $f(x) \neq 0$

⑨ If $h(x) = (f \circ g)(x)$, then

A. $h'(x) = f'(g(x))$. B. $h'(x) = (f \circ g)(x)g'(x)$.

C. $h'(x) = f'(g(x))g'(x)$. D. $h'(x) = f(x)g(x)g'(x)$.

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⑩ Find the derivatives of each of the following functions.

a. $f(x) = 5x^2 + 2x$

b. $f(x) = 3x^3 - \frac{1}{2}x + 5$

c. $f(x) = x^6 + 7x^2 - 5x - 1$

d. $f(x) = 6\sqrt{x} + 3$

e. $f(x) = \frac{2}{x} - \frac{5}{x^2}$

f. $f(x) = 2\sqrt{x} - 6x + 7x^3$

g. $f(x) = x(x^2 + 1)$

h. $f(x) = (x + 3)(4x^4 - 2)$

i. $f(x) = \frac{1}{(x + 2)}$

j. $f(x) = \frac{(2x + 1)}{(x^2 - 1)}$

k. $f(x) = \frac{x^3}{(4 - x^2)}$

l. $f(x) = (x - 5)^2(x + 2)^3$

m. $f(x) = \sqrt{x^2 + 3x}$

n. $f(x) = 3(\sqrt{x} - 5)^2 + 1$

⑪ Determine the slope of the tangent to each of the curves at the given point.

a. $f(x) = \frac{x^3}{5 - 3x^2}$ (1,0.5)

b. $f(x) = (x - 2)^2(x + 1)^3$ (-2,-16)

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⑫ Complete the table with the derivatives.

$f(x) = x^2$ $g(x) = 2x + 1$ $h(x) = \sqrt{x - 3}$

	$y = f(x)h(x)$	$y = \frac{g(x)}{f(x)}$	$y = (h \circ g)(x)$
y'			
y''			

⑬ Find the point(s) where the tangent to the curve is horizontal.

a. $f(x) = -3x^2 + 108x - 722$ b. $f(x) = (x^2 + x + 1)(x - 1)$

⑭ Find the absolute maximum and minimum for each function with the given interval.

a. $f(x) = (x^2 + 3)(x + 3)$, $-3 \le x \le 0$ b. $g(x) = \frac{x^2 + 4}{x}$, $1 \le x \le 5$

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Application

⑮ A ball falls from a cliff at a height of 425 m. The ball's height, h, in metres above the ground after t seconds can be modelled by the function $h(t) = 425 - 4.9t^2$. How fast is the ball falling at 4 seconds?

⑯ A 6500-L swimming pool is drained in 20 minutes. The volume of the water that remains in the pool after t minutes is modelled by $V(t) = 6500(1 - \frac{t}{20})^2$, $0 \le t \le 20$. At what rate is the water flowing out of the pool when $t = 5$?

⑰ The position in metres of a car after t minutes can be modelled by $s(t) = (4 + 3t)^2\sqrt{t + 3}$. Determine the velocity of the car, $v(t)$. What is the velocity of the car at 6 minutes?

⑱ The volume of a balloon can be modelled by $V(r) = \frac{4}{3}\pi r^3$, where r is the radius in cm. Assume that the balloon is a perfect sphere. As the balloon is filled up, the radius is $r(t) = 0.5t^2$, $0 \le t \le 8$, where t is the time in seconds. At what rate will the volume of the balloon change with respect to time at 4 seconds?

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