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6

Sinusoidal Functions

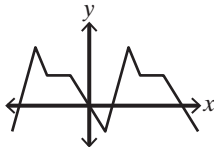


Words TO LEARN

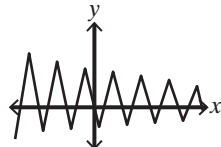
Periodic function:

a function that repeats its values in regular intervals

e.g.



periodic

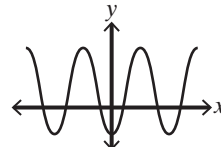


nonperiodic

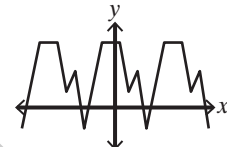
Sinusoidal function:

a periodic function that resembles a smooth curve that is symmetrical

e.g.



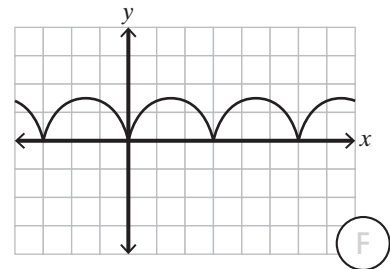
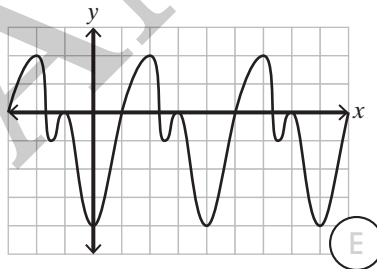
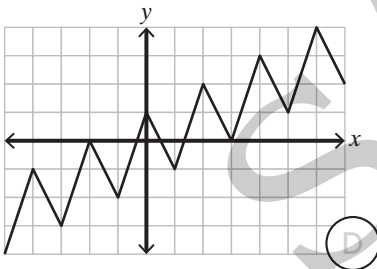
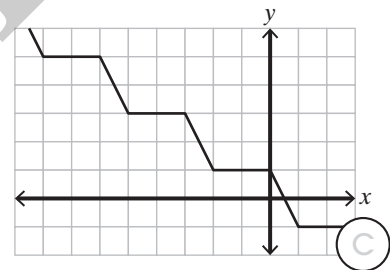
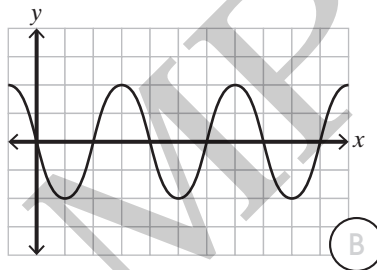
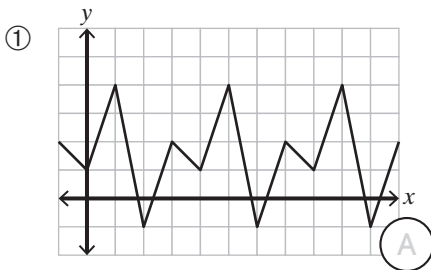
periodic and sinusoidal



periodic but not sinusoidal

6.1 Properties of Periodic Functions

Identify and check the graphs that are periodic. Then answer the question.



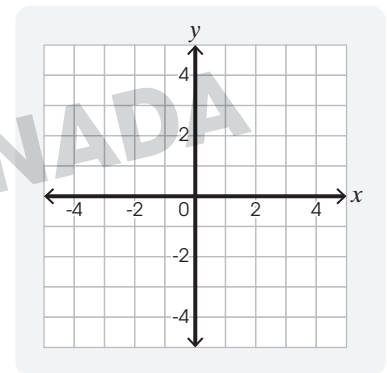
② Which table represents the values of a periodic function? Explain your choice. Then graph the function.

A

x	-5	-4	-3	-2	-1	0	1	2	3	4
y	2	0	-2	4	2	0	-2	4	2	0

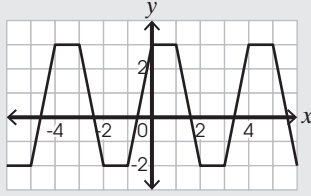
B

x	-5	-4	-3	-2	-1	0	1	2	3	4
y	4	3	2	1	0	-1	-2	-3	-4	-5



Example

Determine the key features of the periodic graph.



Solution:

period: 4 ← The graph repeats its cycle every 4 units on the x -axis.

peak: 3 ← the maximum y -value

trough: -2 ← the minimum y -value

range: $\{y \in \mathbb{R} \mid 3 \geq y \geq -2\}$ ← all possible values of y

equation of the axis: $y = \frac{3 + (-2)}{2}$
 $y = 0.5$

amplitude: $\frac{3 - (-2)}{2} = 2.5$ ← can also be determined using the function's axis
 $3 - 0.5 = 2.5$ or $0.5 - (-2) = 2.5$

Try This

period: _____

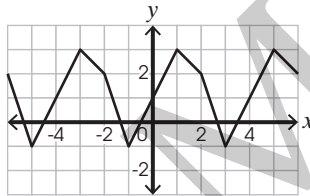
peak: _____

trough: _____

range: _____

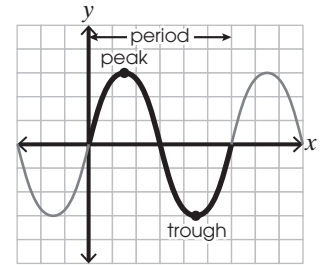
equation of the axis: _____

amplitude: _____



HINT

Key Features of Periodic Functions



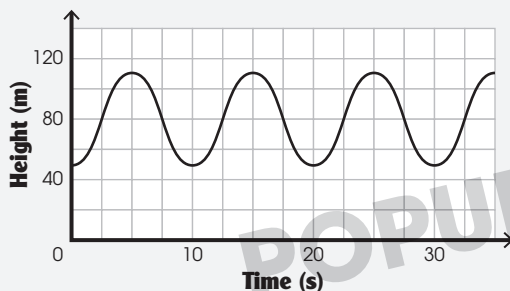
- **period:** the horizontal distance needed for the graph of a periodic function to complete one cycle
- **peak:** the maximum point on a graph
- **trough:** the minimum point on a graph
- **range:** the set of all y -values
- **equation of the axis:** the equation of the horizontal line halfway between the maximum and minimum values

$$y = \frac{\text{max. value} + \text{min. value}}{2}$$
- **amplitude:** half the difference between the maximum and minimum values; or the vertical distance from the function's axis to the maximum or minimum value

Determine the key features of the periodic graphs.

③

Height of a Blade on a Wind Turbine



period: _____

range: _____

peak: _____

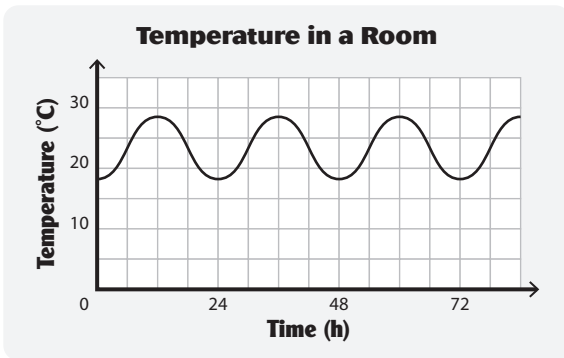
trough: _____

equation of the axis: _____

amplitude: _____

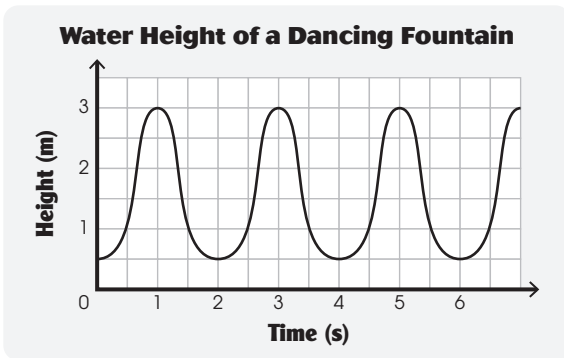


④



period: _____ | range: _____
 peak: _____ | _____
 trough: _____ | equation of the axis: _____
 amplitude: _____ | _____

⑤

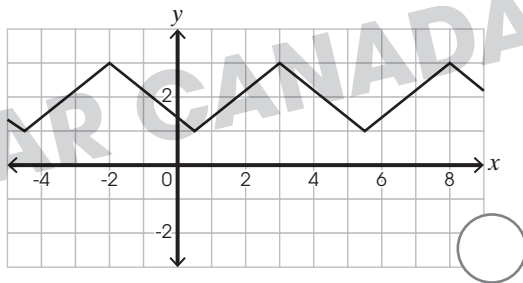
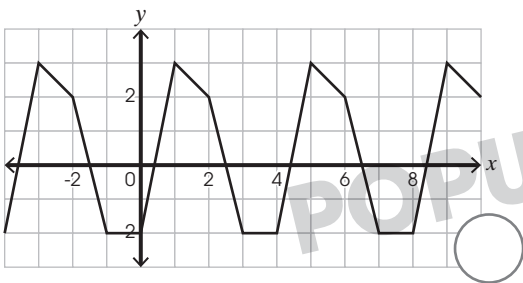
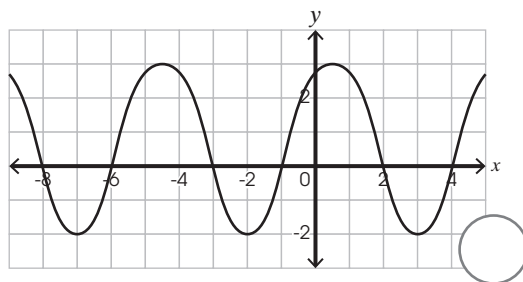
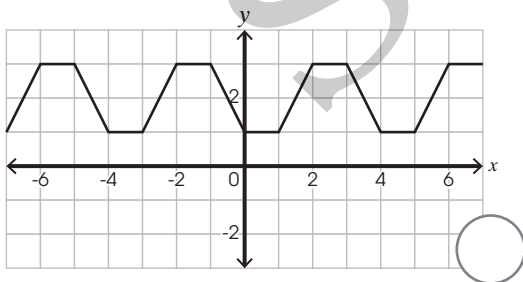


period: _____ | range: _____
 peak: _____ | _____
 trough: _____ | equation of the axis: _____
 amplitude: _____ | _____

Complete the table. Then match the graphs.

⑥

Graph	A	B	C	D
period	4	4	5	5
peak	3	3	3	3
amplitude	_____	1	_____	2.5
equation of the axis	$y = 0.5$	_____	$y = 2$	_____



Determine whether each scenario will produce a periodic or a nonperiodic graph. For the periodic graphs, determine the independent and dependent variables.

⑦ May is skipping with a jump rope.

_____ • independent variable: _____
 _____ • dependent variable: _____

⑧ The water level of a bay is changing due to tides.

_____ • independent variable: _____
 _____ • dependent variable: _____

⑨ Kobe is swimming and he wants to find out the distance he has swum.

_____ • independent variable: _____
 _____ • dependent variable: _____



HINT

An independent variable is a variable whose values are chosen and is usually presented on the x -axis.

A dependent variable is a variable whose values are calculated and is usually presented on the y -axis.

e.g. Jody is jogging.

- independent variable: time
- dependent variable: distance jogged

A table of values is given for each scenario. Plot the points and answer the question.

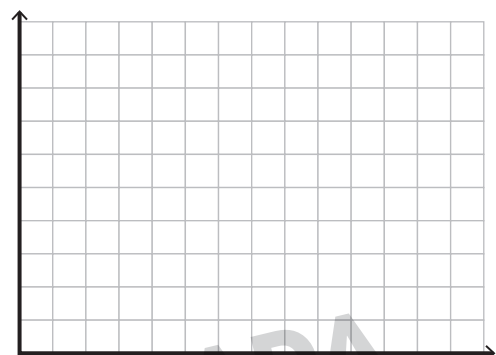
⑩ Water is pumped into and removed from a tube. The table records the amount of water in the tube at specific times.

Time (min)	0	1	2	3	4	5	6	7
Amount (mL)	200	250	250	190	200	250	250	200



Joshua is on a Ferris wheel. His height above the ground over time is recorded in the table.

Time (s)	0	15	30	45	60	75	90
Height (m)	2	8	2	8	2	8	2



⑪ Which graph is periodic? Find its period, peak, trough, range, equation of the axis, and amplitude.
