Contents

Chapter 2

2.2 Laws of Exponents

Key Ideas

Laws of exponents are important because they offer us a quick way to perform operations to simplify expressions.

Laws of Exponents

For $a \neq 0$, p is an integer and m and n are natural numbers.

Product of Powers	$a^m \times a^n = a^{m+n}$	← same base
Quotient of Powers	$a^m \div a^n = a^{m-n}$	← same base
Power of a Power	$(a^m)^n = a^{mn}$	
Negative Exponent	$a^{-p} = \frac{1}{a^p}$	
Identity Exponent	$a^1 = a$	
Zero Exponent	$a^0 = 1$	

Examples

$4^3 \times 4^2$

$= 4 \times 4 \times 4 \times 4 \times 4$

$= \underline{4^5}$

$4^3 \times 4^2$ ← product of powers

$= 4^{3+2}$ ← adding the exponents

$= \underline{4^5}$

$4^5 \div 4^2$

$= \dfrac{4 \times 4 \times 4 \times \cancel{4}^{1} \times \cancel{4}^{1}}{\cancel{4}_{1} \times \cancel{4}_{1}}$

$= \underline{4^3}$

$4^5 \div 4^2$ ← quotient of powers

$= 4^{5-2}$ ← subtracting the exponents

$= \underline{4^3}$

$(4^3)^2$

$= 4^3 \times 4^3$

$= 4 \times 4 \times 4 \times 4 \times 4 \times 4$

$= \underline{4^6}$

$(4^3)^2$ ← power of a power

$= 4^{3 \times 2}$ ← multiplying the exponents

$= \underline{4^6}$

Try these!

Fill in the blanks.

① $a^m \times a^n = a^{m+n}$

a. $5^3 \times 5^2 = 5^{\square + \square}$

$= 5^{\square}$

b. $7^2 \times 7^4 = \square^{2+4}$

$= \square^{6}$

② $a^m \div a^n = a^{m-n}$

a. $5^3 \div 5^2 = 5^{\square - \square}$

$= 5^{\square}$

b. $2^6 \div 2^2 = \square^{6-2}$

$= \square^{4}$

③ $(a^m)^n = a^{mn}$

a. $(2^3)^5 = 2^{\square \times \square}$

$= 2^{\square}$

b. $(3^4)^2 = \square^{4 \times 2}$

$= \square^{8}$

④ $a^{-p} = \frac{1}{a^p}$

a. $5^{-2} = \dfrac{1}{5^{\square}}$

b. $3^{-6} = \dfrac{1}{\square^{6}}$

⑤ $a^1 = a$

a. $7^1 = \square$

b. $6^1 = \square$

⑥ $a^0 = 1$

a. $5^0 = \square$

b. $4^0 = \square$

Practice

Fill in the blanks.

⑦ a. $3^2 \times 3^5 = 3^{\square + \square} =$ _____

b. $4^8 \div 4^6 = 4^{\square - \square} =$ _____

c. $(6^3)^4 = 6^{\square \times \square} =$ _____

d. $(4^2)^5 = 4^{\square \times \square} =$ _____

e. $3^{-2} = \frac{1}{3^{\square}}$

f. $4^0 =$ _____

g. $5^1 =$ _____

h. $2^{-5} =$ _____

i. $9^0 =$ _____

j. $2^0 =$ _____

k. $4^{-3} =$ _____

l. $3^{-4} =$ _____

Evaluate each multiplication or division of powers. Write the answer as a single power. Show your work.

⑧ $3^2 \times 3^4$

= __________

= ________

⑨ $5^3 \times 5^2$

⑩ $6^4 \times 6^2$

⑪ $8^4 \div 8^3$

⑫ $10^5 \div 10^3$

⑬ $10^5 \div 10^1$

⑭ $2^0 \times 2^7$

⑮ $7^9 \div 7^0$

Write each answer as a single power.

⑯ $(2^3)^2 =$ _____

⑰ $(4^2)^3 =$ _____

⑱ $(8^5)^2 =$ _____

⑲ $(10^2)^2 =$ _____

⑳ $(3^2)^4 =$ _____

㉑ $(8^0)^7 =$ _____

㉒ $(9^2)^5 =$ _____

㉓ $(4^6)^5 =$ _____

Express each answer as a power with a positive exponent.

㉔ 2^{-2}

㉕ 5^{-2}

㉖ 10^{-3}

㉗ 6^{-2}

㉘ 3^{-3}

㉙ $\left(\frac{1}{3}\right)^{-2}$

㉚ $\left(\frac{1}{5}\right)^{-3}$

㉛ $\left(\frac{2}{3}\right)^{-2}$

Hint

$\left(\frac{1}{2}\right)^{-2} = \left(\frac{2}{1}\right)^2 = 2^2$

$\frac{2}{1}$ is the reciprocal of $\frac{1}{2}$.

Chapter 2

2.4 Squares and Square Roots

Key Ideas

Squares are powers with an exponent of 2. When a number is raised to the power of 2, we say the number is squared. On the other hand, square roots are the opposite of squares.

Square Root Rules

Consider $a \geq 0$.

- $\sqrt{a} \times \sqrt{a} = a$
- $\sqrt{a^2} = a$

Consider $a, b \geq 0$.

- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, where $b \neq 0$
- If $a^2 = b$, then $\sqrt{b} = a$.

Examples

$\underset{3}{\underset{\uparrow}{\sqrt{9}}} \times \underset{3}{\underset{\uparrow}{\sqrt{9}}} = 9 \leftarrow \sqrt{a} \times \sqrt{a} = a$

$\underset{\sqrt{81}}{\underset{\uparrow}{\sqrt{9^2}}} = 9 \leftarrow \sqrt{a^2} = a$

$\underset{6}{\underset{\uparrow}{\sqrt{36}}} = \underset{2}{\underset{\uparrow}{\sqrt{4}}} \times \underset{3}{\underset{\uparrow}{\sqrt{9}}} \leftarrow \sqrt{ab} = \sqrt{a} \times \sqrt{b}$

$\underset{\frac{3}{2}}{\underset{\uparrow}{\sqrt{\frac{9}{4}}}} = \underset{\frac{3}{2}}{\underset{\uparrow}{\frac{\sqrt{9}}{\sqrt{4}}}} \leftarrow \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

$4^2 = 16$, so $\sqrt{16} = 4 \leftarrow \begin{matrix} a^2 = b \\ \sqrt{b} = a \end{matrix}$

Try these!

Fill in the blanks.

① $\sqrt{a} \times \sqrt{a} = a$

a. $\sqrt{4} \times \sqrt{4} = \square$

b. $\sqrt{10} \times \sqrt{10} = \square$

② $\sqrt{a^2} = a$

a. $\sqrt{16^2} = \square$

b. $\sqrt{20^2} = \square$

③ $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

a. $\sqrt{9 \times 16} = \sqrt{9} \times \sqrt{\square}$

b. $\sqrt{25 \times 2} = \sqrt{25} \times \sqrt{\square}$

④ $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

a. $\sqrt{\frac{100}{4}} = \frac{\sqrt{100}}{\sqrt{\square}}$

b. $\sqrt{\frac{16}{4}} = \frac{\sqrt{\square}}{\sqrt{4}}$

⑤ If $a^2 = b$, then $\sqrt{b} = a$.

a. $5^2 = 25$

$\sqrt{25} = \square$

b. $4.5^2 = 20.25$

$\sqrt{20.25} = \square$

c. $\left(\frac{3}{4}\right)^2 = \frac{9}{16}$

$\sqrt{\frac{9}{16}} = \square$

Evaluate each square root without using a calculator.

⑥ $\sqrt{5^2}$ = ________ ⑦ $\sqrt{8} \times \sqrt{8}$ = ________ ⑧ $\sqrt{10^2}$ = ________

⑨ $\sqrt{7} \times \sqrt{7}$ = ________ ⑩ $\sqrt{12^2}$ = ________ ⑪ $(\sqrt{3.5})^2$ = ________

⑫ $\sqrt{4 \times 4}$ = ________ ⑬ $\sqrt{2} \times \sqrt{32}$ = ________ ⑭ $\sqrt{4.5 \times 2}$ = ________

⑮ $\sqrt{\frac{36}{9}}$ = ________ ⑯ $\sqrt{\frac{8}{0.5}}$ = ________ ⑰ $\frac{\sqrt{18}}{\sqrt{2}}$ = ________

Simplify the square roots. Show your work.

Hint

To simplify a square root, first rewrite the number within the root as a product of the largest perfect square possible and another number. Then find the square roots to simplify.

e.g. $\sqrt{75}$

$= \sqrt{25 \times 3}$ ← 25 is the largest perfect square possible.

$= \sqrt{25} \times \sqrt{3}$

$= 5\sqrt{3}$

⑱ $\sqrt{12}$

$= \sqrt{\boxed{} \times 3}$

$= \sqrt{\boxed{}} \times \sqrt{3}$

$= \boxed{}\sqrt{3}$

⑲ $\sqrt{20}$

⑳ $\sqrt{27}$

㉑ $\sqrt{50}$

㉒ $\sqrt{32}$ ㉓ $\sqrt{54}$ ㉔ $\sqrt{48}$

㉕ $\sqrt{72}$ = ____________________ ㉖ $\sqrt{120}$ = ____________________

㉗ $\sqrt{150}$ = ____________________ ㉘ $\sqrt{200}$ = ____________________

㉙ $\sqrt{243}$ = ____________________ ㉚ $\sqrt{363}$ = ____________________